Solutions of Mock JEE Advanced-4 (CBT) | Paper - 2 | 2024

PHYSICS

SECTION-1

1.(B) $dV = -E \cdot dr$

$$\int_B^A dV = -\int_{(0,2,4)}^{(2,1,0)} (x\hat{i} - 2y\hat{j} + z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$\therefore \quad V_A - V_B = -\int_{(0,2,4)}^{(2,1,0)} (xdx - 2ydy + zdz)$

Or $\quad V_{AB} = -\left[\frac{x^2}{2} - y^2 + \frac{z^2}{2}\right]_{(0,2,4)}^{(2,1,0)}$

$= 3$ volts

2.(D) $[LT^{-1}] = [MC^2T^{-1}]^a [T^{-1}]^b [MT^{-3}]$

$\Rightarrow \quad a + c = 0 \quad \ldots.(i)$

$2a = 1 \quad \ldots. (ii)$

$-a - b - 3c = -1 \quad \therefore \quad a = \frac{1}{2}, c = \frac{-1}{2}, b = 2$

3.(B) $a = \frac{dv}{dt} = (8\hat{i} - 4t\hat{j})$

At 1 s $\quad F_{\text{net}} = ma = (1)(8\hat{i} - 4\hat{j}) = (8\hat{i} - 4\hat{j}) = W + F$

where, F = force on cube

$\therefore \quad F = (8\hat{i} - 4\hat{j}) - w = (8\hat{i} - 4\hat{j}) - (-10\hat{j}) = (8\hat{i} + 6\hat{j})$ Or $|F| = \sqrt{(8)^2 + (6)^2} = 10N$

4.(B) $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 273}{28 \times 10^{-3}}}$ = 493 m/s

Now, increase in gravitation PE = decrease in kinetic energy

$\therefore \quad \frac{(mg\,h)}{1 + \frac{h}{R}} = \frac{1}{2} m v_{\text{rms}}^2$ or $\frac{2gh}{1 + (h/R)} = v_{\text{rms}}^2$ Or $\frac{2 \times 9.81 \times h}{1 + (h/6400 \times 10^3)} = (493)^2$

Solving this equation, we get $h \approx 12000\,\text{m}$ Or 12 km

SECTION-2

5.(AC) $d\sin\theta = \eta \quad \Rightarrow \quad d\tan\theta \approx \eta$

$d\frac{y}{D} \approx \eta \quad \Rightarrow \quad y = \frac{\eta D}{d}$

$\therefore \quad y = \frac{600 \times 10^9 \times 2}{10^{-3}} = 1.2\,mm \quad \therefore \quad 2y = 2.4\,mm$

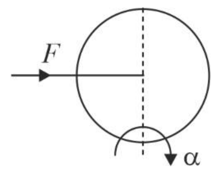
Now, $\tan\theta = \pm\frac{\eta}{d}$

$= \frac{600 \times 10^{-9}}{10^{-3}} = 6 \times 10^{-4} \quad \therefore \quad \Delta\theta = 12 \times 10^{-4}$

6.(ABCD)

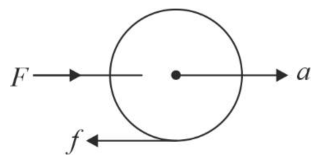
(A) Bottom most point where friction actually acts is at rest.

(B) $\alpha = \dfrac{F \cdot R}{\dfrac{3}{2}mR^2} = \dfrac{2F}{3mR}$

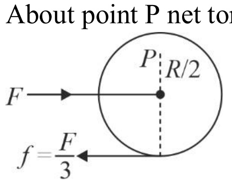


$a = R\alpha = \dfrac{2F}{3m}$

$F - f = ma = \dfrac{2F}{3}$ $\quad\therefore\quad$ $f = \dfrac{F}{3} = \dfrac{1}{2}ma$



(C) About point P net torque of F and *f* is zero. So, angular momentum is conserved.



(D) Torque about bottom-most point $= Fr = \dfrac{dL}{dt} \Rightarrow L = Frt$

7.(ABCD)

$f = 10^{15}, \lambda = 10^{-7} \Rightarrow C =$ speed in medium $= 10^8$

$\Rightarrow$ Refractive index of medium $= \dfrac{3\times10^8}{10^8} = 3$

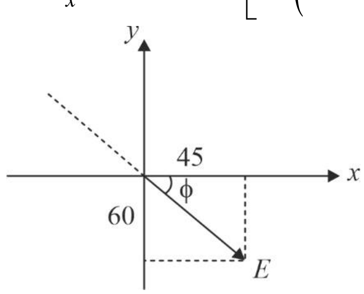
$\# \hat{E} \equiv \left(\dfrac{3}{5}\hat{i} - \dfrac{4}{5}\hat{j}\right)$: direction of propagation $= \hat{e} \equiv \hat{k}$

$\Rightarrow \hat{B} = \hat{e} \times \hat{E} = \hat{k} \times \left(\dfrac{3}{5}\hat{i} - \dfrac{4}{5}\hat{j}\right) = \left(\dfrac{4}{5}\hat{i} + \dfrac{3}{5}\hat{j}\right) \# B_{\max} = \dfrac{E_{\max}}{C} = \dfrac{75}{10^8} = 7.5\times10^{-7}\, Wb/m^2$

$\Rightarrow \vec{B} = 7.5\times10^{-7}\left(\dfrac{4}{5}\hat{i} + \dfrac{3}{5}\hat{j}\right)\sin\left[2\pi\left(10^{15}t - 10^7 z\right)\right]$

$\Rightarrow \vec{B} = 1.5\times10^{-7}\left(4\hat{i} + 3\hat{j}\right)\sin\left[2\pi\left(10^{15}t - 10^7 z\right)\right]$

$\Rightarrow B_x = 6\times10^{-7}\sin\left[2\pi\left(10^{15}t - 10^7 z\right)\right] \Rightarrow B_y = 4.5\times10^{-7}\sin\left[2\pi\left(10^{15}t - 10^7 z\right)\right]$



$\tan\phi = \dfrac{60}{45} = \dfrac{4}{3} \Rightarrow \theta = 53^\circ$

SECTION-3

1.(1) $\# \; J = mv$

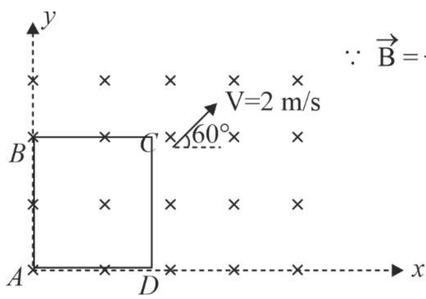
$$\# Jr = \frac{ml^2}{12}\omega$$

$$\# T = \frac{2V}{g} = \frac{2J}{mg}$$

$$\# \theta = \omega T = \frac{12Jr}{ml^2} \cdot \frac{2J}{mg} = \frac{24J^2 r}{m^2 l^2 g} = 2\pi n$$

$$\Rightarrow \quad n = \frac{12J^2 r}{\pi m^2 l^2 g} = \frac{12 \times \frac{\pi}{2} \times 10^{-4} \times 0.04}{\pi \times 240 \times 10^{-6} \times 0.01 \times 10} = 1$$

2.(2)



$$\because \; \vec{B} = \frac{-B_0}{L} x \; \hat{k}$$

$\because$ In the loop, motional emf developed is only due to side CD, effectively.

$$E_{eq} = \frac{B_0 a}{L}.aV_x \Rightarrow i = \frac{E_{eq}}{R} = \frac{B_0 a^2}{RL}.v_x = \frac{2 \times 10^{-5}}{10} = 2 \times 10^{-6} \text{ amperes.}$$

3.(750) $SOD - SD = \lambda$

Where, $\lambda = \frac{v}{f} = \frac{360}{360} = 1m \quad \therefore \quad 2\sqrt{\frac{x^2}{4} + (2)^2} - x = \lambda = 1$

Solving this equation, we get $\quad x = 7.5m$

4.(5) $\sigma e \, A\left(T^4 - T_0^4\right) = mS\left(\frac{dT}{dt}\right) \quad \therefore \quad \frac{dT}{dt} = \frac{\sigma e A\left(T^4 - T_0^4\right)}{mS} = \frac{\sigma e\left(4\pi \frac{d^2}{4}\right)\left(T^4 - T_0^4\right)}{\rho \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 S}$

On putting values and solving, we get $\frac{dT}{dt} = 200 \times 10^{-3} {}^\circ C / s$

5.(32) Activity of S_1 after n_1 half-lives $= A_1 = \frac{A_{10}}{2^{n_1}}$

Activity of S_2 after n_2 half-lives $= A_2 = \frac{A_{20}}{2^{n_2}}$

Then $\frac{A_1}{A_2} = \frac{A_{10}}{A_{20}} \cdot 2^{(n_2 - n_1)} = \frac{A_{10}}{A_{20}} \cdot 2^{(7-4)} = 8\frac{A_{10}}{A_{20}} = 8 \times 4 = 32$

6.(2) Process 1 – 2 and 3 – 4 are isochoric process.

$\therefore \quad W_{12} = W_{34} = 0$

2 – 3 and 4 – 1 are isobaric processes.

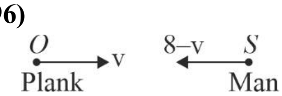
$\therefore \quad W_{23} = P_2 \Delta V = P_2 V_3 - P_2 V_2 = nRT_3 - nRT_2 = nR(T_3 - T_2)$

$W_{41} = P_1 \Delta V = P_1 V_1 - P_1 V_4 = nR(T_1 - T_4) = -nR(T_4 - T_1)$

$\therefore \quad W = nR[T_3 - T_2 - T_4 + T_1] \quad \Rightarrow \quad W = nR[6T_1 - 2T_1 - 3T_1 + T_1] = 2nRT_1$

SECTION-4

7.(9296)



From conservation of linear momentum,

$$50(8 - v) = 150v$$

$\therefore \quad v = 2\,\text{m/s}$

$8 - v = 6\,\text{m/s}$

$$f_1 = f_0\left(\frac{v + v_0}{v - v_s}\right) = f_0\left(\frac{330 + 2}{330 - 6}\right) = \frac{332}{324} f_0 = \frac{332}{324} \times 9072 = 9296\,Hz$$

8.(8856)

$$f_2 = f_0\left(\frac{v - v_0}{v + v_s}\right) = f_0\left(\frac{330 - 2}{330 + 6}\right) = \left(\frac{328}{336}\right) f_0 = \frac{328}{336} \times 9072 = 8856\,Hz$$

9.(141.42) For iso-thermal changes,

PV = constant Or $\frac{P}{\rho} =$ constant

Between point A and point B, (inside the funnel)

$$\frac{P_A}{\rho_A} + O + \frac{V_A{}^2}{2} = \frac{P_B}{\rho_B} + gh + \frac{V_B{}^2}{2} \qquad \because \quad \frac{P_A}{\rho_A} = \frac{P_B}{\rho_B}$$

$\Rightarrow \quad V_A^2 = V_B^2 + 2gh \qquad \because \quad V_B = 0 \Rightarrow V_A = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 10^3} = 141.42\,\text{m/sec}$

10.(1.35) $P_A = P_B + \rho_{cold}.g.h$ (hydrostatic from outside the funnel)

$\because \quad PM = \rho RT$

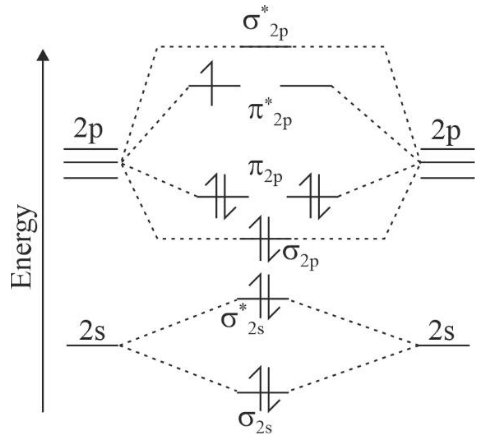
$$\Rightarrow \quad \frac{\rho_{Hot} RT_{hot}}{M} = \frac{\rho_{cold} RT_{cold}}{M} + \rho_{cold}.g.h \quad \Rightarrow \quad \rho_{Hot} RT_{hot} = \rho_{cold} T_{cold} + \left(\frac{Mgh}{R}\right)\rho_{cold}$$

$$\Rightarrow \quad \rho_{cold} = \frac{\rho_{Hot} T_{Hot}}{T_{Cold} + \frac{Mgh}{R}} = \frac{1.3 \times 320}{273 + \frac{30 \times 10^{-3} \times 10 \times 10^3}{8.3}} \approx 1.35\,\text{kg/m}^3$$

CHEMISTRY

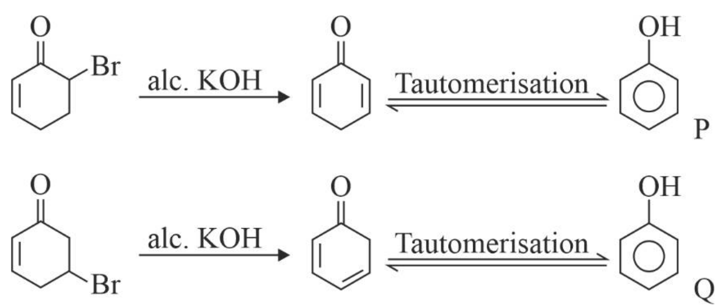
SECTION-1

1.(B) The correct MO diagram for O_2^+ is given.

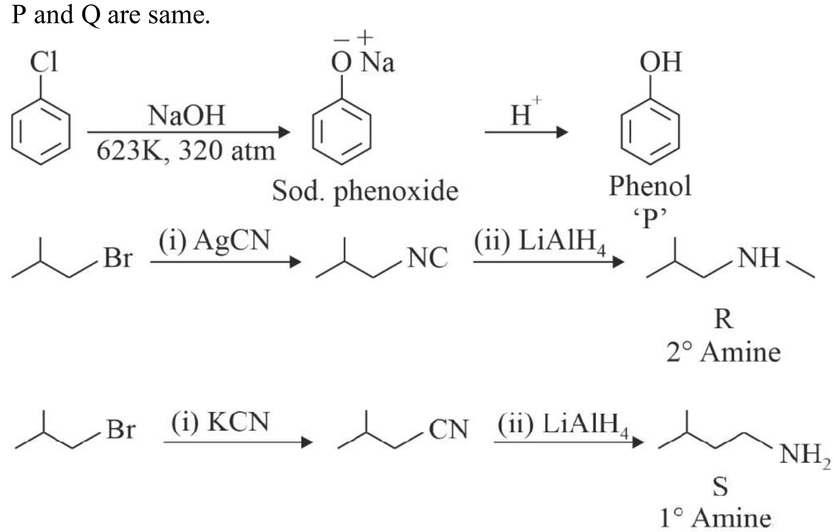


2.(C) (II) Whipped cream is a foam, which is a gas dispersed in a liquid
(IV) lyophilic sols are also named as reversible sols.

3.(B)



P and Q are same.



R and S are functional isomers.

4.(A) Compound given in A is sucrose i.e., a disaccharide. Sucrose is a non-reducing sugar and does not give Fehling's test and Tollen's test as it does not contain hemiacetal group.
All other disaccharides are reducing sugars.

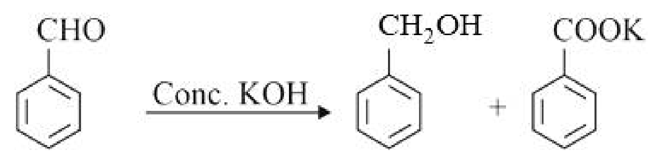
SECTION-2

5.(AD)

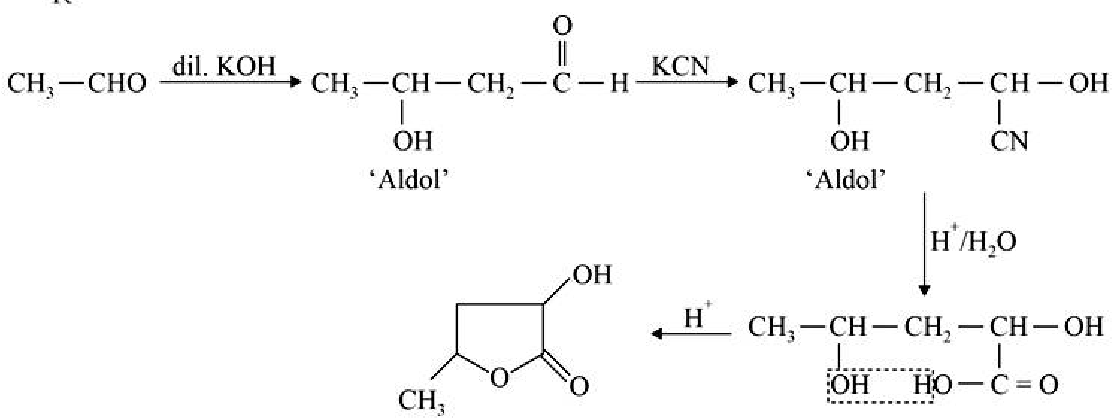
$[Cu(CN)_4]^{3-}$	:	Diamagnetic with d^{10} configuration.
$[Fe(CN)_6]^{4-}$	:	Diamagnetic with Fe in +2 O.S.
$[Cu(NH_3)_4]^{2+}$	:	Paramagnetic with d^9 configuration
$[Fe(H_2O)_5NO]^{2+}$	:	Paramagnetic with Fe in +1 O.S.
$[Fe(CN)_5NO]^{2-}$	:	Diamagnetic with Fe in +2 O.S.

6.(ACD)

$$Ph-CH=CH-CH_3 \xrightarrow[2.\ Zn]{1.\ O_3} \underset{Q}{CH_3-CHO} + \underset{R}{Ph-CHO}$$



'R'



(S)

7.(AB)

(A) Effective no. of C-atoms per unit cell = 8

mass $= 12 \times 8 = 96$ amu

(B) $\frac{\sqrt{3}a}{4} = B.L_{C-C}$

$$D = \frac{4 \times 154}{\sqrt{3}} = 355.66 pm$$

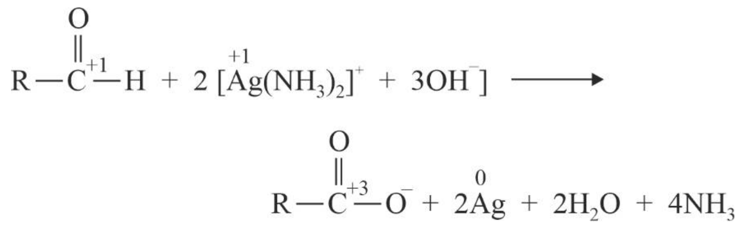
(C) $D = \frac{ZM}{N_A a^3} = \frac{8 \times 12}{6 \times 10^{23} \times (355.66 \times 10^{-10})^3}$

$= 3.56 g/cm^3$

(D) For every carbon-atom, coordination number = 4

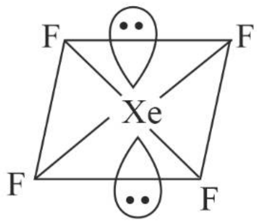
SECTION-3

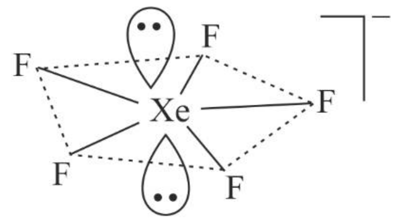
1.(2)

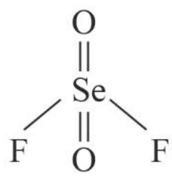


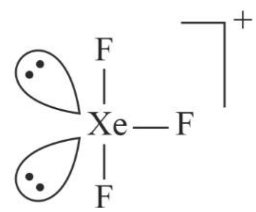
No. of electrons transferred = 2

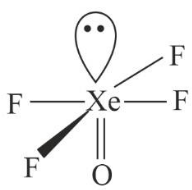
2.(6)

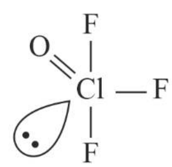
XeF_4


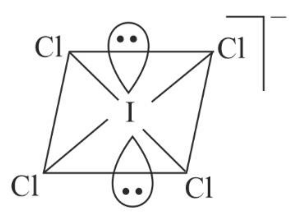
XeF_5^-


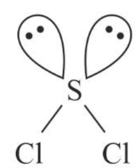
F_2SeO_2


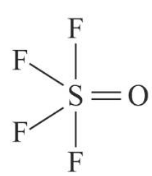
XeF_3^+


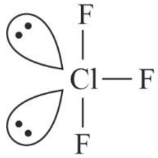
$XeOF_4$


$ClOF_3$


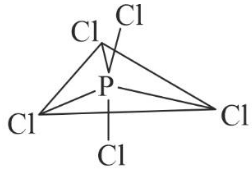
ICl_4^-


SCl_2


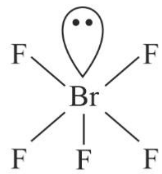
OSF_4




ClF_3



PCl_5



BrF_5

3.(14) 1. $[\overset{x_1}{Fe}_2(CO)_9]$

$2x_1 + 9(0) = 0$

$x_1 = 0$

2. $[\overset{x_2}{Fe}(NH_3)_6]SO_4$

$x_2 + 6\times(O) = +2$

$x_2 = 2$

3. $Na[\overset{x_3}{Fe}(NH_3)_2(CN)_4]$

$+1 + x_3 + 2(0) + 4\times(-1) = 0$

$x_3 = +3$

4. $K_3[\overset{x_4}{Fe}(CN)_6]$

$3\times(+1) + x_4 + 6\times(-1) = 0$

$x_4 = +3$

5. $Na_4[\overset{x_5}{Fe}(CN)_5(NOS)]$

$4(+1) + x_5 + 5(-1) + (-1) = 0$

$x_5 = +2$

6. $Na_4[\overset{x_6}{Fe}O_4]$

$4(+1) + x_6 + 4(-2) = 0$

$x_6 = +4$

(sum of oxidation state of iron atoms) $\rightarrow$ x

$x = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 0 + 2 + 3 + 3 + 2 + 4 = 14$

4.(8) x = 8

We have $\Delta E = \frac{3}{4}\times 0.85\text{eV}$

Energy = 0.6375

Energy of the photon corresponds to Brackett series

For Brackett $0.31 \le E \le 0.85$

$$0.85\times\left(1-\frac{1}{4}\right) = 13.6\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$$

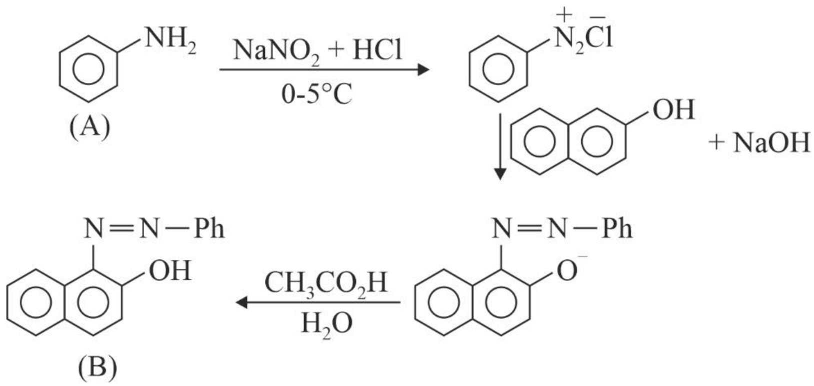
$$0.85\left(1-\frac{1}{4}\right) = \frac{13.6}{16}\left[1-\left(\frac{4}{n}\right)^2\right] \therefore \quad \frac{4}{n} = \frac{1}{2} \Rightarrow n = 8 \qquad \text{Hence, x = 8}$$

5.(4) $\Delta T_f = K_f \cdot m$; $\Delta T_f = 6.49 - 6.30 = 0.19$

$$0.19 = 4.9\times\frac{\frac{0.2}{M}}{\frac{17.2}{1000}} \Rightarrow 0.19 = 4.9\times\frac{1000\times 0.2}{M\times 17.2} \Rightarrow M = \frac{4.9\times 1000\times 0.2}{17.2\times 0.19} = 300$$

Atomicity $= \frac{300}{75} = 4$.

6.(25)

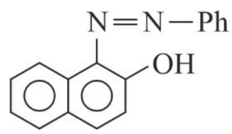


Molecular weight of aniline = Molecular weight of C_6H_7N

Density of $A = 1.2 \text{ gm mL}^{-1}$

15.5 mL of A = 18.6 gm A = $\frac{18.6}{93}$ = 0.2 mole A

The mole ratio

= $PhNH_2 : PhN_2^+ :$ 

= 1 : 1 : 1

The moles of Q formed = 0.2 mole [If extent of reaction is 100%]
For 50% yield.

Amount of Q formed $= 0.2 \times \frac{50}{100} = 0.1$ mol

The molecular formula of Q = $C_{16}H_{12}ON_2$

So, molecular weight of $Q = 16 \times 12 + 12 \times 1 + 16 + 2 \times 14 = 192 + 12 + 16 + 28 = 248$ gm

So, amount of $Q = 248 \times 0.1 = 24.8$ gm

SECTION-4

7.(4050) From graph, slope = –1

$\therefore$ eq $= \log_{10}V + \log_{10}T = \text{constant}$

$\therefore VT = \text{constant}$ or PV^2 = constant

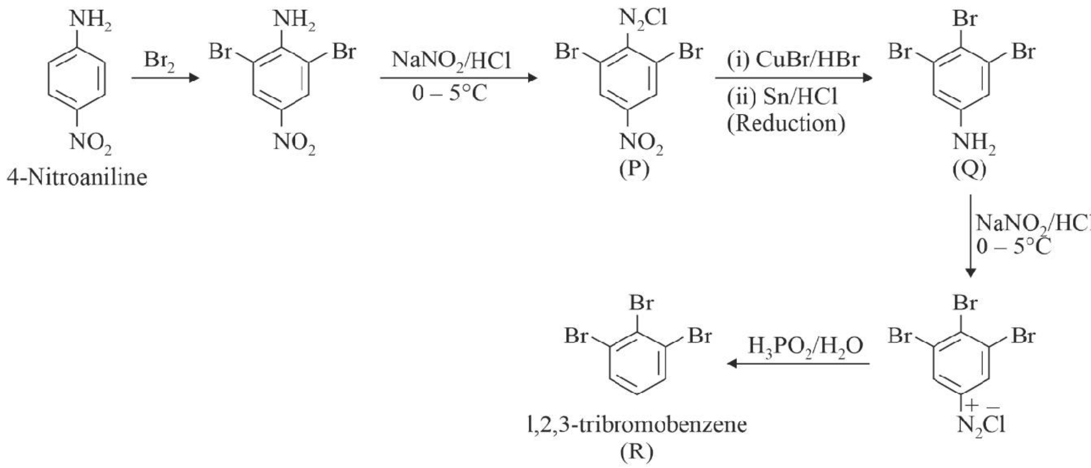
$C = C_V + \frac{R}{1-n}$ $\quad\quad \therefore C = \frac{5R}{2} + \frac{R}{1-2} = \frac{3R}{2}$

$\therefore q = nC\Delta T = 3 \times \frac{3R}{2} \times (1000 - 100) = 4050\,R$

8.(2700) $\Delta U = nC_V\Delta T = 3 \times \frac{5R}{2} \times (1000 - 100) = 6750\,R$

$\therefore W = \Delta U - q = 6750\,R - 4050R = 2700R$

9.(315)



Molecular formula of R is $C_6H_3Br_3$

Molar mass of $R = 6\times 12 + 3\times 1 + 3\times 80$

$= 72 + 3 + 240$

$= 315$

10.(10) No. of C atoms in Q = 6

No. of Hetero atoms in Q = 4

The total number of carbon atoms and heteroatoms present in one molecule of Q is 10.

MATHEMATICS

SECTION-1

1.(B) $p(n)$ = Probability that child stops after drawing exactly n marbles

$$p(n)=\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.......\left(\frac{n-1}{n}\right)\left(\frac{1}{n+1}\right)=\frac{1}{n(n+1)}$$

2.(C) $e^{xy}\left[xy^2+y^3\frac{dx}{dy}\right]+e^{\frac{x}{y}}\left[y\frac{dx}{dy}-x\right]=0$; $d\left(e^{xy}\right)+d\left(e^{\frac{x}{y}}\right)=0$

$e^{xy}+e^{\frac{x}{y}}+c=0$; $A+B=2$

3.(A) If $0<x<1, a=6$; if $x>1, a=2$, if $x<-1, a=-2$ only and if $-1<x<0, a=2$

4.(C) Given $\vec{c}\times\vec{b}=2\vec{a}\times\vec{b}\Rightarrow(\vec{c}-2\vec{a})\,||\,\vec{b}\Rightarrow\vec{c}=2\vec{a}+\mu\vec{b}\Rightarrow\lambda=2$ ...(1)

Also from (1) $\Rightarrow\vec{a}.\vec{c}=2+\mu\vec{a}.\vec{b}=2+\mu\frac{1}{4}$ …(2)

Again, from (1) $\Rightarrow\vec{b}.\vec{c}=2\vec{a}.\vec{b}+\mu=\frac{1}{2}+\mu$ …(3)

Again, from (1) $\Rightarrow\vec{c}.\vec{c}=2(\vec{a}.\vec{c})+\mu(\vec{b}.\vec{c})\Rightarrow16=2.(2+\frac{\mu}{4})+\mu\left(\frac{1}{2}+\mu\right)$ {from (2) and (3)}

$$\Rightarrow12=\frac{\mu}{2}+\frac{\mu}{2}+\mu^2\Rightarrow\mu^2+\mu-12=0$$

SECTION-2

5.(ABC) $f(x).f(y)+f(x+y)=e^x f(y)+e^y(x)+xy.$

$\therefore f(x).f'(y)+f'(x+y)=e^x f'(y)+e^y f(x)+x$

(Differentiate both side w.r.t. y treating x as constant).

Put $y=0$

$f(x).f'(0)+f'(x)=e^x f'(0)+f(x)+x\Rightarrow f'(x)-f(x)=x\Rightarrow f(x)=e^x-x-1$

$\therefore(A)\ \lim\limits_{x\to0}\frac{f(x)}{x^2}=\lim\limits_{x\to0}\frac{e^x-x-1}{x^2}=\frac{1}{2}.$

(B) $\int\limits_x^{x^2}t\,dt=\left(\frac{t^2}{2}\right)_x^{x^2}=\frac{x^4-x^2}{2}>0\,\forall\,|x|>1$

(C) $F(x)=f'(x)-f(x)=\left(e^x-1\right)-\left(e^x-x-1\right)=x$ which is increasing function so

$F(x_2)>F(x_1)$ if $x_2>x_1$

(D) Now $f'(x)=e^x-1$ and $f''(x)=e^x>0\,\forall x\in R$

$\therefore f'(x)$ is increasing function on R

$\because f'(-1)=-ve$ and $f'(1)=+ve$

$\therefore f'(c)=0$ has exactly are root in (-1, 1).

i.e,, one horizontal tangent in (–1, 1)

6.(AC) $A^3+B^3+C^3 = A^3+B^3+C^3-3ABC$

$= (A+B+C)(A^2+B^2+C^2-AB-BC-CA)$

$= (A+B+C)(A+\omega B+\omega^2 C)(A+\omega^2 B+\omega C)$ [ω = cube root of unity]

$= (A+B+C)(A+\omega B+\omega^2 C)\overline{(A+\omega B+\omega^2 C)}$

thus $\det(A^3+B^3+C^3).\det(A+B+C)$

$\det(A+B+C)^2.\det(A+\omega B+\omega^2 C)\det\overline{(A+\omega B+\omega^2 C)}$

$= \det(A+B+C)^2.\left|\det(A+\omega B+\omega^2 C)\right|^2 \ge 0$

7.(AB) $\int_0^1 g(x)dx = \frac{11}{6} \Rightarrow \frac{a}{3}+\frac{b}{2}+c = \frac{11}{6} \Rightarrow 2a+3b+6c=11$

$\therefore a=b=c=1\{\because a,b,c \in N$

$\because \frac{f(x)f'(x)}{\sqrt{1-f(x))^4}} - x \ge 0 \Rightarrow h(x) = \sin^{-1}(f(x))^2 - x^2$ is non-decreasing function

$\therefore \lim_{x\to x_1^+} h(x) \le \lim_{x\to x_2^+} h(x) \Rightarrow \frac{\pi}{2}-x_1^2 \le \frac{\pi}{6}-x_2^2 \Rightarrow x_1^2-x_2^2 \ge \frac{\pi}{3} \Rightarrow \left[x_1^2-x_2^2\right] \ge 1$

i.e., $\left[x_1^2-x_2^2\right]=1$.

SECTION-3

1.(96) $P_1 = \frac{9!}{4!3!2!}.\frac{1}{3^9}$ (4 must go in B_1, 3 in B_2 and 2 in B_3)

and $P_2 = \frac{9!3!}{(3!)^3 3!}.\frac{1}{3^9}$ all boxes are different $\therefore \frac{P_2}{P_1} = \frac{4}{3}$

2.(8) $\because$ Elements of determinant can be 1 or -1

$\because$ First two places of first row can be filled in 2 × 2 = 4 ways and last place can be filled in only one way to product be 1

$\therefore$ First row can be made in 4 ways

Similarly, second row can be made in 4 ways and third row can be made in 1 ways only

$\therefore m = 16$

$$\because |P| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & -2 & 3 \\ 0 & -1 & 1 \end{vmatrix} = 1$$

$\because \text{adj}(\text{adj}P) = |P|^{n-2} P = P \Rightarrow \text{tr}(\text{adj}(\text{adj}P)) = \text{tr}(P) = 2 = n.$

$\therefore \frac{m}{n} = \frac{16}{2} = 8$

3.(10) $f'(x)=3\left(x^2-2x-3\right)=3(x+1)(x-3)$

Case-I $f'(x)>0$ i.e. $f(x)$ is increasing the $x>3$ or $x<-1$

$f(f(x))<f(x^3-4x^2-3x+19)\Rightarrow f(x)<x^3-4x^2-3x+19$

$\Rightarrow x^2-6x+8<0\Rightarrow x\in(2,4)$

$\therefore x\in(3,4)$

Case-II $f'(x)<0$ i.e, $f(x)$ is decreasing then $-1<x<3$

$f(x)>x^3-4x^2-3x+19$

$\Rightarrow x^2-6x+8>0\Rightarrow (x-2)(x-4)>0\Rightarrow x<2$ or $x>4$

$\therefore x\in(-1,2)$

$\therefore x\in(-1,2)\cup(3,4)$

$\therefore b-a+c+d=2+1+3+4=10$

4.(1) $g(f(x))=x$

So, $g'(f(x))\cdot f'(x)=1$

$$g''(f(x))=-\frac{f''(x)}{\left(f'(x)\right)^3} \qquad \text{...(i)}$$

$$\because e^{-x}f(x)=\frac{3}{e^2}+4e^{-x}\int_2^x\sqrt{2t^2+6t+5}\,dt \qquad \text{...(A)}$$

Differentiate both sides w.r.t. to x, we get

$$-e^{-x}.f(x)+e^{-x}.f'(x)=-4e^{-x}\int_2^x\sqrt{2t^2+6t+5}\,dt+4e^{-x}\sqrt{2x^2+6x+5}$$

From (A)

$$-e^{-x}f(x)+e^{-x}.f'(x)=\frac{3}{e^2}-e^{x}.f(x)+4e^{-x}\sqrt{2x^2+6x+5}$$

$$\Rightarrow e^{-x}.f'(x)=\frac{3}{e^2}+4e^{-x}\sqrt{2x^2+6x+5}$$

$$\Rightarrow f'(x)=\frac{3}{e^2}.e^x+4\sqrt{2x^2+6x+5} \qquad \text{...(B)}$$

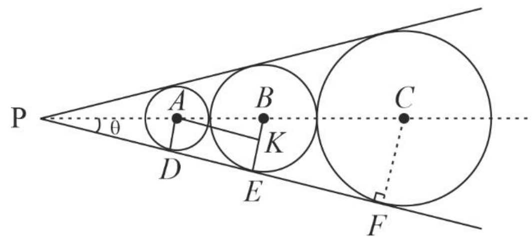
$$f''(x)=\frac{3}{e^2}.e^x+\frac{4(2x+3)}{\sqrt{2x^2+6x+5}} \qquad \text{...(C)}$$

Put $x=2$ in (A), (B) and (C), we get

$$f(2)=3, f'(2)=23, f''(2)=\frac{43}{5}$$

So, $g'(3)=\dfrac{1}{23}$ and $g''(3)=\dfrac{-43}{5(23)^3}\Rightarrow\left[g'(3)\right]+\left[\left|g''(3)\right|\right]=1$

5.(4)



$DE = AK = \sqrt{(r_2+r_1)^2-(r_2-r_1)^2} = 2\sqrt{r_1r_2}$

Similarly, $EF = 2\sqrt{r_2r_3}$

$\angle APD = \theta$ and PD = x then $\tan\theta = \frac{AD}{PD} = \frac{BE}{PE} = \frac{CF}{PF} = \frac{r_1}{x} = \frac{r_2}{x+2\sqrt{r_1r_2}} = \frac{r_3}{x+2\sqrt{r_1r_2}+\sqrt{r_2r_3}}$

Hence, $\frac{r_2-r_1}{2\sqrt{r_1r_2}} = \frac{r_3-r_2}{2\sqrt{r_2r_3}} \quad \Rightarrow r_2 = \sqrt{r_1r_3} \Rightarrow r_2 = \sqrt{2\times 8} = 4$

6.(5) Assume $\alpha_i < \frac{\pi}{6}$

$2(\sin\alpha_1 + \sin\alpha_2 + \sin\alpha_3) = 3\sin(\alpha_1+\alpha_2+\alpha_3)$

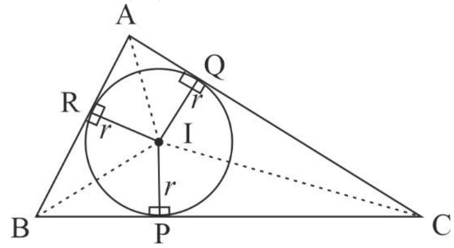
Also, $\frac{\sin\alpha_1+\sin\alpha_2+\sin\alpha_3}{3} \le \sin\left(\frac{\alpha_1+\alpha_2+\alpha_3}{3}\right)\left[\alpha_i \in \left(0,\frac{\pi}{2}\right)\right]$

$\Rightarrow \frac{\sin(\alpha_1+\alpha_2+\alpha_3)}{2} \le \sin\left(\frac{\alpha_1+\alpha_2+\alpha_3}{3}\right) \Rightarrow \sin 3t \le 2\sin t \;\Rightarrow 4\sin^3 t - \sin t \ge 0$

$\Rightarrow \sin^2 t \ge \frac{1}{4}$, which contradicts assumption $\Rightarrow \arg(z_1) = \arg(z_2) = \arg(z_3) = \frac{\pi}{6}$

SECTION-4

7.(2) **8.(27)**



Let $\tan\frac{A}{2} = x, \tan\frac{B}{2} = y, \tan\frac{C}{2} = z$

$\therefore x = \frac{r}{AR}, y = \frac{r}{BP}, z = \frac{r}{CQ}$

Put the value of *AR, BP* and *CQ* in the given relation, we get

$$\frac{2x}{r} + \frac{5y}{r} + \frac{5z}{r} = \frac{6}{r}$$

$\Rightarrow \quad 2x+5y+5z = 6 \qquad \ldots(1)$

Also, in the triangle

$xy+yz+zx = 1 \qquad \ldots(1)$

If we interchange y and z, then both equations (i) and (ii) remains unchanged

$\Rightarrow$ ΔABC is isosceles Δ with $\angle B = \angle C \Rightarrow y = z$.

So, from (i) and (ii), we get

$x = 3 - 5y$

and $2xy + y^2 = 1$

Solving we get, $x = \frac{4}{3}$ and $y = z = \frac{1}{3}$.

$\therefore AR = \frac{3r}{4}, BP = CQ = 3r$

Now perimeter $2s = 2.AR + 2.BP + 2CQ = \frac{27r}{2}$

It is given that perimeter is smallest integer.

$\therefore$ $r = 2$

and $s = \frac{27}{2}$

and Area of $\Delta ABC = \Delta = rs$

$= 2 \times \frac{27}{2} = 27$

9.(128)

X	P(X)	x.P(X)
0	$P(0) = 0$	0
1	$P(1) = 0$	0
2	$P(2) = \frac{4}{81}$	$\frac{8}{81}$
3	$P(3) = \frac{28}{81}$	$\frac{84}{81}$
4	$P(4) = \frac{49}{81}$	$\frac{196}{81}$

$E(X) = \frac{288}{81} \Rightarrow 36.E(x) = \frac{288}{81} \times 36 = 128$

10.(1.60) Two consecutive points can be chosen in

$2 \times 9 \times 8 = 18 \times 8$ ways

$\therefore n(E) = 144$ and $n(s) = {}^{81}C_2$

$\therefore p(E) = \frac{144}{{}^{81}C_2} = \frac{2}{45}$

$\therefore 36.p(E) = 36 \times \frac{2}{45} = \frac{8}{5} = 1.60$